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THE LIGHT, THE HEAVY AND THE SUPERHEAVY

— A NONABELIAN FLAVOR SYMMETRY FOR THE FULL HIERARCHY*

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ABSTRACT

We give a preliminary report of a new quark mass matrix model basing on a $SU(5) \otimes SU(5) \otimes Q_{12}$ symmetry embedding into a fully gauged $SU(5) \otimes SU(5) \otimes SU(2)$.¹ The two $SU(5)$'s contain the standard SUSY $SU(5)$ as a diagonal subgroup, while the Q_{12} or $SU(2)$ is horizontal. Starting by assuming a judiciously-chosen set of chiral supermultiplets, and a pattern of spontaneous symmetry breaking, we obtain the low-energy chiral fermions together with a spectrum of superheavy fermions at two different scales. The latter mediate Froggatt-Nielsen tree graphs that give rise to a phenomenologically viable effective quark mass matrix texture. The model is the first example of a nontrivial combination of supersymmetry without R-parity, gauged nonabelian horizontal symmetry and unification/anti-unification. It is expected to have some very interesting features in SUSY-GUT phenomenology.

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1. The Light, the Heavy and the Superheavy

The smallness of most of the quark mass and mixing parameters and the strong hierarchy among them is one of the most interesting puzzle in particle physics. Flavor symmetry, especially a horizontal symmetry commuting with the Standard Model group or the GUT group, paired with the Froggatt-Nielsen mechanism^{2,3}, provide a plausible explanation for the hierarchical texture pattern⁴ of the quark mass matrices. Following this popular approach, here we present a new model using a discrete dicyclic subgroup⁵, Q_{12} , of a gauged horizontal $SU(2)$. The model is the first example of a nontrivial combination of supersymmetry without R-parity, gauged nonabelian horizontal symmetry and unification/anti-unification⁶.

We recall the hierarchy in the quark sector parameters given in powers of $\lambda \sim 0.22$, at around the GUT scale:^{3,4}

$$\begin{aligned} |V_{us}| &\sim \lambda, & |V_{cb}| &\sim \lambda^2, & |V_{ub}| &\sim \lambda^3 - \lambda^4; \\ m_u/m_c &\sim \lambda^3 - \lambda^4, & m_c/m_t &\sim \lambda^3 - \lambda^4, \\ m_d/m_s &\sim \lambda^2, & m_s/m_d &\sim \lambda^2, \\ m_b/m_t &\sim \lambda^3, & m_t/\langle H_u \rangle &\sim 1. \end{aligned}$$

To construct the light and heavy quark masses, the Froggatt-Nielsen mechanism invokes a spectrum of superheavy fermions, which are essentially in vector-like pairs, to communicate the effects of the flavor symmetry breaking vevs to the low-energy chiral fermions. Integrating out the superheavy particles then leaves us with effective quark mass terms containing powers of small parameters of the form $\frac{\langle S \rangle}{M}$, say $\sim \lambda$ or λ^2 , where $\langle S \rangle$ is a symmetry breaking vev and M the superheavy fermion mass allowed by the unbroken symmetry.

2. The $SU(5) \otimes SU(5) \otimes SU(2)$ Model

The model has a gauge symmetry given by $SU(5) \otimes SU(5) \otimes SU(2)$. For the two $SU(5)$'s, they are the GUT groups for the third family and the lighter two families respectively. The latter form a horizontal doublet.

2.1. Symmetry Breaking Pattern

The two $SU(5)$'s then break into a diagonal $SU(5)$ which is identified as the standard unification group, at energy scale M_0 . So below M_0 is the standard SUSY-GUT story. We assume that the horizontal $SU(2)$ is broken to Q_{12} at energy scale $M_{Q_{12}}$ with the latter subsequently totally broken at around M_0 . The symmetry

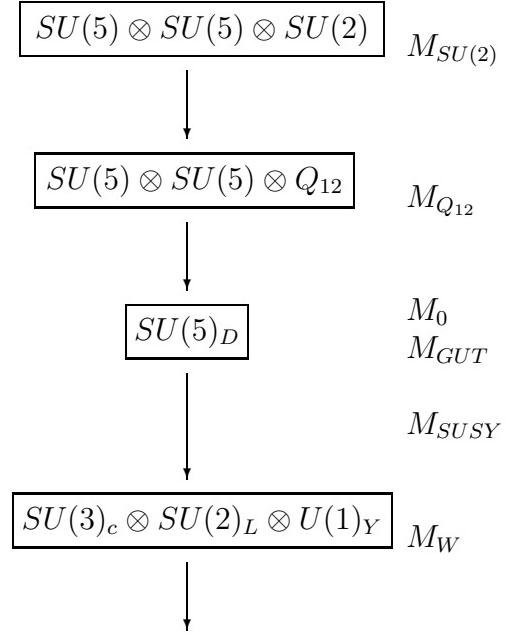


Fig. 1. The Symmetry Breaking Pattern of the Model

breaking pattern is summarised in Figure 1.

2.2. Fermion Content

The light, heavy and superheavy fermion contents of the model come from the following list of $SU(5) \otimes SU(5)$ chiral supermultiplets:-

<i>from</i>	$SU(2)$	<i>to</i>	Q_{12}
$(10, 1) -$	1	\longrightarrow	$1(T)$
	7	\longrightarrow	$1' + 2_2 + 2_4 + 2_6$
	4	\longrightarrow	$2_1 + 2_3$
$(\bar{10}, 1) -$	7	\longrightarrow	$1' + 2_2 + 2_4 + 2_6$
	4	\longrightarrow	$2_1 + 2_3$
$(\bar{5}, 1) -$	6	\longrightarrow	$2_1 + 2_3 + 2_5(H_d/b)$
	3	\longrightarrow	$1' + 2_2$
$(5, 1) -$	1	\longrightarrow	$1(H_u)$
	4	\longrightarrow	$2_1 + 2_3$
	3	\longrightarrow	$1' + 2_2$
$(1, 10) -$	2	\longrightarrow	$2_1(Q)$
	1	\longrightarrow	1
$(1, \bar{10}) -$	1	\longrightarrow	1
$(1, \bar{5}) -$	2	\longrightarrow	$2_1(D)$

(A summary of the Q_{12} representations is given in the appendix.)

The vector-like fermion pairs have Dirac masses of order $M_{SU(2)}$ where $SU(2)$ is a good symmetry. At $M_{Q_{12}}$, the two $(5, 1)$ doublets, 2_1 and 2_3 from the dimension 4 $SU(2)$ representation married with the corresponding $(\bar{5}, 1)$ doublets from the dimension 6 $SU(2)$ representation to form Dirac fermions, leaving behind only the labelled Q_{12} singlets and doublets as GUT scale chiral particles. The list of low energy chiral fermions is given by

$$\begin{array}{ccc} \left(\begin{array}{c} u \\ d \\ c \\ s \end{array} \right)_L & Q(2_1) & \left. \begin{array}{c} u_L^c \\ c_L^c \\ d_L^c \\ s_L^c \end{array} \right\} \quad \begin{array}{c} Q(2_1) \\ D(2_1) \end{array} \\ \left(\begin{array}{c} t \\ b \end{array} \right)_L & T(1) & \left. \begin{array}{c} t_L^c \\ b_L^c \end{array} \right\} \quad \begin{array}{c} T(1) \\ H_d/b(2_5) \end{array} \end{array}$$

and the correspondent leptonic partners of the GUT multiplets, and the Higgsinos from $H_u = 1$ and $H_d/b = 2_5$. Recall that in the $SU(5)$ language, H_u is a 5, Q and T are 10's while D and the interesting H_d/b are $\bar{5}$'s.

The horizontal doublet H_d/b is definitely the most interesting element of the model. It contains both the bottom-tau and the (down-sector) Higgs chiral multiplets. The group properties of the representation 2_5 plays a very important role in the model, as discussed below.

2.3. Supersymmetry without R-parity

The most interesting point to note about the assignment of the bottom-tau and down-sector Higgs to a horizontal doublet is that it is incompatible with the standard R-parity, which is put into MSSM by hand to avoid unacceptable B and L violation. Here in our model, the group properties of the horizontal symmetry gives this required matter parity feature⁷.

The only direct Yukawa couplings to Higgses are for the third family, giving rank one quark mass matrices for both up and down sector at the first order. In terms of MSSM chiral superfields, the top get its mass from the term $\hat{q}\hat{h}_1\hat{u}^c$ contained in the 10.5.10 coupling (Figure 2a). The bottom and the tau get their masses from the terms $\hat{q}\hat{h}_2\hat{d}^c$ and $\hat{l}\hat{h}_2\hat{e}^c$ respectively, both contained in the 10.5.5 coupling (Figure 3a). The latter however does not give rise to the dangerous terms $\hat{q}\hat{l}\hat{d}^c$, $\hat{l}\hat{l}\hat{e}^c$, and $\hat{u}^c\hat{d}^c\hat{d}^c$. The *secret* is in the Q_{12} product $2_5 \times 2_5$ which contains the singlet 1 only in the antisymmetric part, therefore always coupling the bottom-tau part to the Higgs part but not to itself.

Moreover, no $\bar{5}.5$ term is allowed by the horizontal symmetry, hence both $\hat{l}\hat{h}_1$ and $\hat{h}_2\hat{h}_1$ are absent. In conclusion, to the first order, the horizontal symmetry gives the

required matter parity feature and evades the μ -problem. Detailed properties of the model in aspects like B and L violation, FCNC, and Higgsino masses and the role of the horizontal symmetry in them is a very interesting question to be addressed.

2.4. The Quark Mass Matrices

All the above listed chiral supermultiplets are assumed to develop no vevs above the M_{GUT} . We need then the following vector-like multiplet with the specified vevs to generate the effective quark mass matrices. They are, a $<2_4>$ of $(1, 1)$, a $<2_1>$ of $(1, 24)$, a $<2_1>$ and a $<2_2>$ of $(\bar{10}, 10)$ coupled only to 2nd family in Q or D , a $<2_2>$ of $(5, \bar{5})$ coupled only to 2nd family in Q or D , and a $<2_1>$ of $(5, \bar{5})$ coupled only to 1st family in Q or D .

The model is now completed. The lowest order effective quark mass term for the up and down sector are given by the Froggatt-Nielsen tree graphs shown in Figure 2 and 3 respectively. Taking

$$<2_k>/M_{Q_{12}} \sim \lambda, \quad <2_k>/M_{SU(2)} \sim \lambda^2; \quad <2_k>\sim M_0,$$

we arrive at the quark mass textures

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_d \sim \begin{pmatrix} 0 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{pmatrix}.$$

A comparison with the corresponding symmetric texture pattern from reference 4, as given by

$$M_u \sim \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_d \sim \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{pmatrix}.$$

is sufficiently convincing that it is phenomenologically viable.

Possible alternative formulation of the model under a $SU(5) \otimes U(1) \otimes SU(2)^1$ and comparison with the simple Q_{2N} models built under the same approach⁸ are skipped here.

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4. Appendix on Q_{2N} Representations^{5,8}

May be a dihedral group D_N as the symmetry of a N-sided planar polygon is more familiar to physicists. The dicyclic group Q_{2N} is a double cover of D_N , as the parent continuous group $SU(2)$ being a double cover of the rotation group $SO(3)$. Q_{2N} is of

order $4N$; Q_{12} , the $N = 6$ candidate in our model has order 24.

Irreducible representations of Q_{2N} are given by 4 singlets $1, 1', 1'', 1'''$ and $(N - 1)$ doublets 2_k (with $1 \leq k \leq (N - 1)$). Most important for our purposes are the product formulae: -

$$1' \times 1' = 1 \quad (1)$$

$$1' \times 2_k = 2_k \quad (2)$$

$$2_k \times 2_l = 2_{(|k-l|)} + 2_{(\min\{k+l, 2N-k-l\})} \quad (3)$$

where, in a generalized notation, $2_0 \equiv 1 + 1'$ and $2_N \equiv 1'' + 1'''$.

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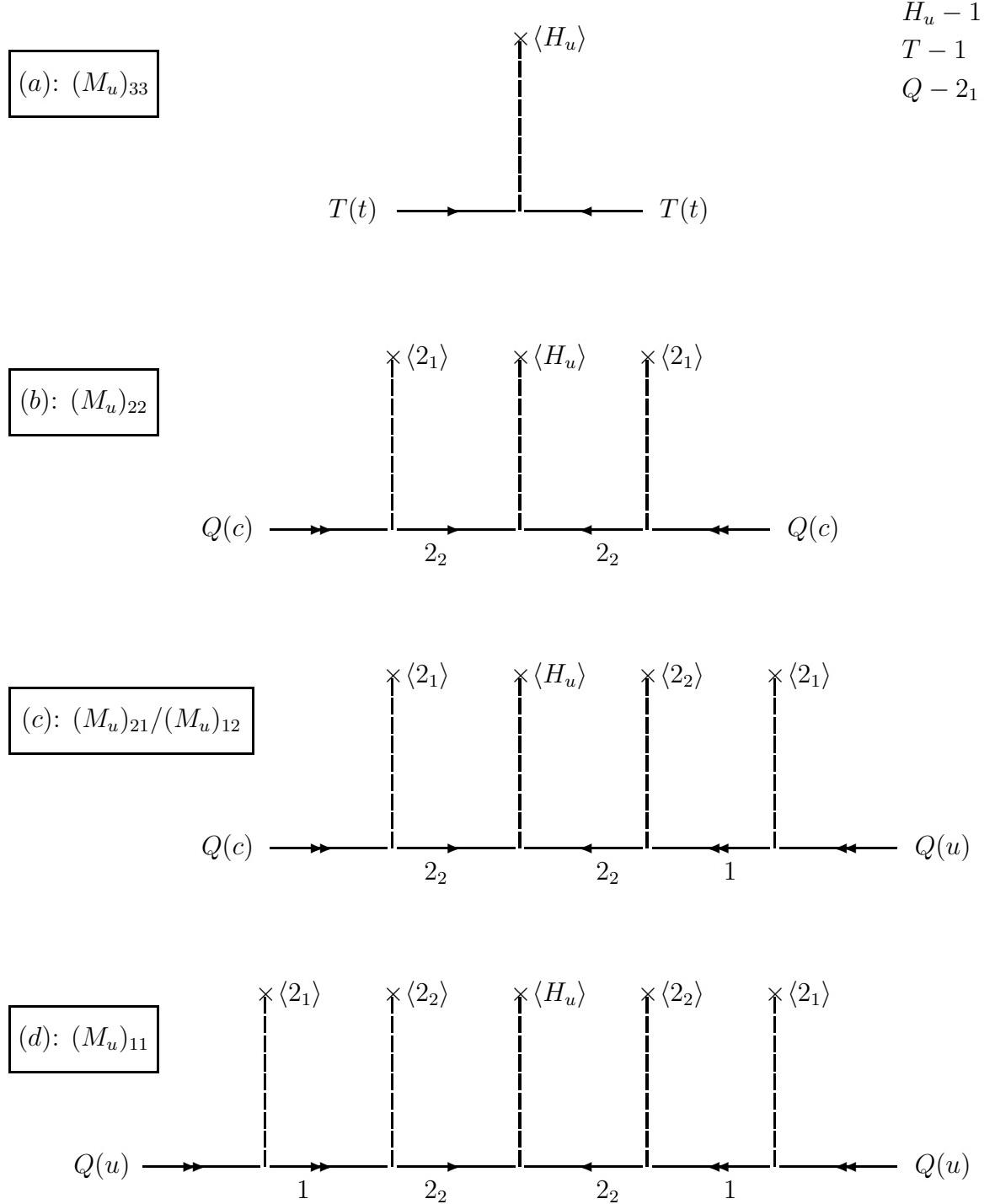
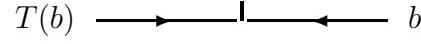


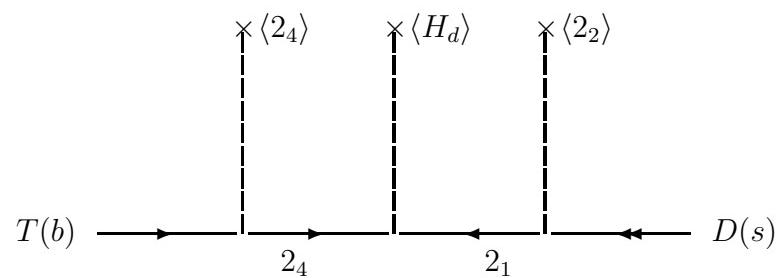
Fig. 2. Froggatt-Nielsen tree graphs for M_u .*

(a): $(M_d)_{33}$

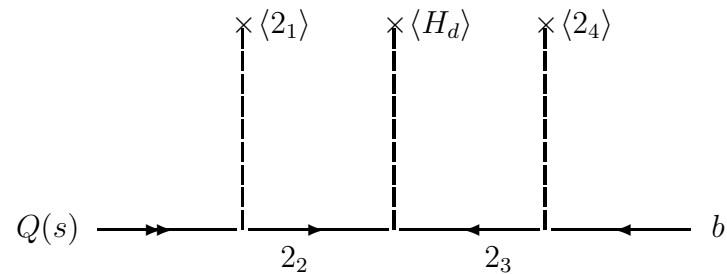
$\times \langle H_d \rangle$	$T - 1$
$\times \langle H_d \rangle$	$H_d/b - 2_5$
$\times \langle H_d \rangle$	$Q - 2_1$
$\times \langle H_d \rangle$	$D - 2_1$



(b): $(M_d)_{32}$



(c): $(M_d)_{23}$



(d): $(M_d)_{13}$

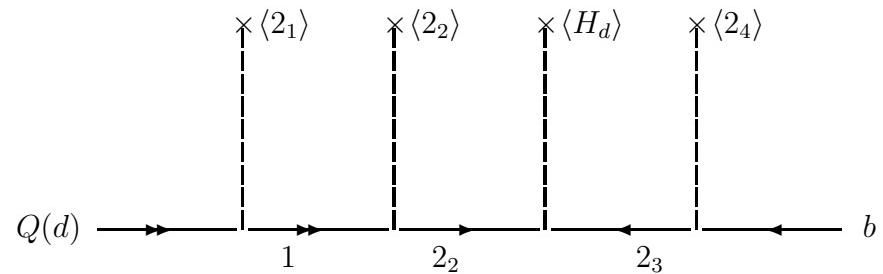


Fig. 3. Froggatt-Nielsen tree graphs for M_d .*

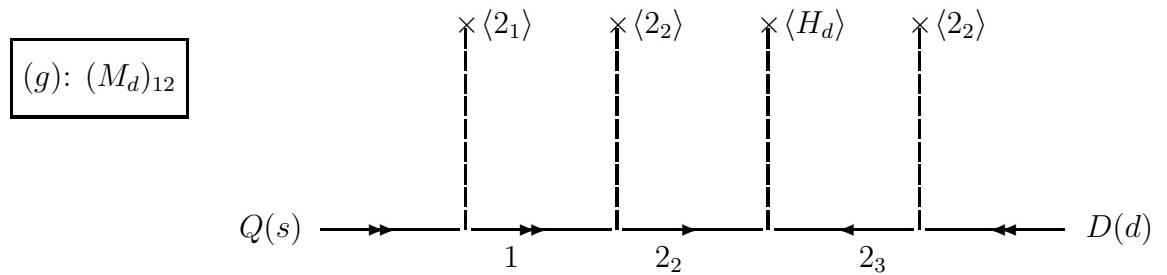
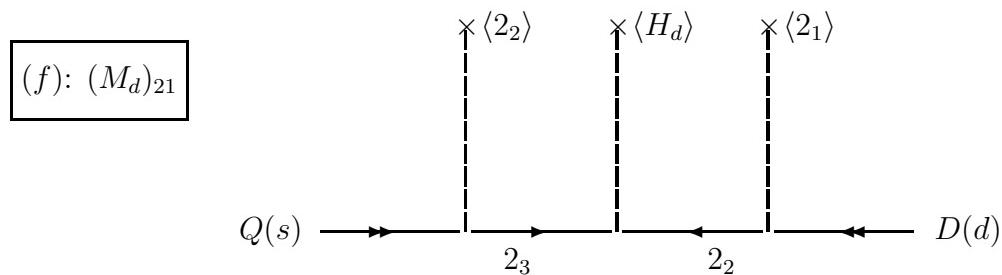
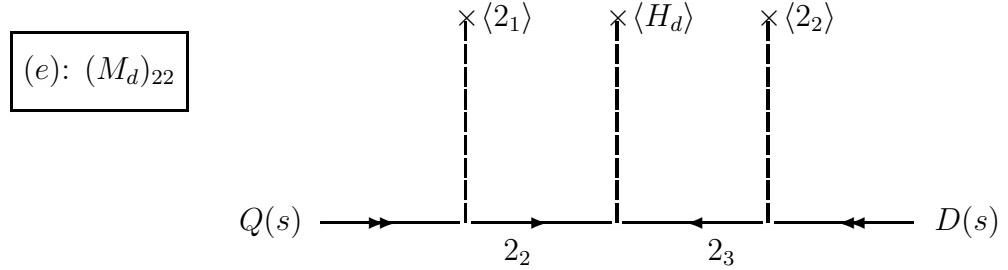


Fig. 3. Froggatt-Nielsen tree graphs for M_d .*

* In both Figure 2 and 3, the superheavy fermions and scalar vevs are indicated by their representations in terms of the Q_{12} symmetry, while the low energy chiral particles are indicated by their label. All vertical dash-lines represent scalar vevs.

In Figure 2, all horizontal lines represent fermions; those shown by arrows are $(10, 1)$'s while those shown by double-arrows are $(1, 10)$'s under $SU(5) \otimes SU(5)$. Likewise in Figure 3, except that those left pointing arrows and double-arrows (fermions to the right of H_d) are here $(\bar{5}, 1)$'s and $(1, \bar{5})$'s respectively. The full representations for the scalars can then be figured out easily.